APPLICATION OF CALCULUS IN ECONOMICS

Dr. Manoj Kr. Mandal
Assistant Professor
Department of Economics
R.D. & D.J. College
Munger University, Munger (Bihar)
Email: drmkmandal02@gmail.com

Abstract

Calculus is used in every branch of statistics, economics, physical sciences, computer science, engineering, medicine, demography and in other fields wherever a problem can be mathematically modeled and an optimal solution is desired.

For a decision-making authority, whose ever it may be (a consumer or a producer) such a question is of vital importance for making final decision or solving any economic problem. In economics, this question is the core of the marginal analysis.

This concept of 'margin' is essentially a mathematical concept. Differential calculus is immensely useful in dealing with problems of change at margin e.g., rates, equilibrium, elasticities, etc. In the reverse process of calculating total from marginal, Integral calculus is of much help.

Keywords

Calculus,consumer, producer, marginal analysis, Differentiation, Integration

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Dr. Manoj Kr. Mandal

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Introduction

Calculus is a branch of mathematics that deals with rates of change. Calculus is concerned with two basic operations, differentiation and integration. Differentiation is used to break down the function into parts and integration is used to unite those parts to form the original function. Therefore, in other words, integration is the reverse process of integration.

If' be a function defined on the closed interval (a,b) Divide this interval into n subinterval by choosing any (n-1) Intermediate points between a and b. Let X0=a and Xn=b and x1<x2<x3………. < xn-1 be the intermediate point such that x0<x1<x2 ………. < xn-1<xn. Let Δ, x be the length of the first subinterval so that Δ1 x=x1–x0 and let $\Delta_2 x$ be the length of the second subinterval so that $\Delta_2\, x=x_2-x_1$ and so on, thus the length of the ith subinterval is Δ| x=xi–xi–1. A set of all such subintervals of the interval [a, b] is called a partition Δ, called the nom of the partition, and is demoted by || Δ ||. Let be the point chosen in (xo-xi) so that x0<x2ε<x_1. Let ε2 the point chosen in $[x_1-x_2]$ so that x1<ε_2 and so on, thus ε_i be the point chosen in $[x_{i-1}-x_i]$ and $x_{i-1}<\varepsilon_i<x_i$. Form the sum-

$$f(\varepsilon_1)\Delta_1 x+f(x_2)\Delta_2 x+f(\varepsilon_3)\Delta_3 x+ \text{--------} +f(\varepsilon_n)\Delta_n x \text{ or,}$$

$$\sum_{i=1}^{n} f(\varepsilon_i)\Delta_i x$$

If f is a function defined on the closed interval $[a,b]$, then

the definite integral of f from a to be, is given by –

$\int_a^b f(x)dx= lim \sum_{i=1} f(\varepsilon_i)\Delta_i x$ if the limit exists. thus in short we have, for a function.

y=f(x) --------------- (1)

Indefinite integration of the function f(x) with respect to x is given by-

$\int f(x)dx+c$ where c is arbitrary constant -------------- (2) and definite integration of the function f(x) with respect to x in the interval $[a,b]$ is given by-

$$\int_a^b f(x)dx \text{ ------------- (3)}$$

If differentiate equation (1) with respect to x, then we get the first derivative of the function (1) which is denoted by $\frac{dy}{dx}$ or f' (x). If we differentiate again $\frac{d}{id}$ or f'(x). Then we get the second-order derivative of the function (1) which is denoted by $\frac{d^2y}{dx^2}$ or f" x. In general, we represent the nth derivative of $f(x)$ as denoted by $f^n(x)$

or $\frac{\square\square}{\square\, Dryden}$ The first-order and second-order derivatives play an important role in finding max. and min. value of the function. This concept is used in economics for determining the maximum profit of a firm. Mathematics provides economic laws and relationships, and makes them more practical. In economics, calculus is used to determine marginal revenue and price which helps to maximize their profit. As long as marginal revenue exceeds the price, the firm increases its profit. In short, the application of integration in economics helps to look out the total cost function and total revenue function from the worth. Economists use the following functions, where,

C(x) = Cost function

R(x) = Revenue function

π(x) =Profit

We know that, $\boxed{\pi (x) = R(x) - C(x)}$

Profit is max when, $\frac{d\pi}{dx} = \frac{dR}{dx} - \frac{dC}{dx} = 0$

So, $\frac{dR}{dx} = \frac{dC}{dx}$

Therefore, MR = MC

The cost function consists of two parts, fixed cost demoted by F and variable cost demoted by V(x). This means that –

$C(x) = F+v(x)$ ------------- (4)

If differentiate (4) with respect to x then we get rate of change of cost C per unit change in the output level of x unit which is called as marginal cost (Mc). Thus,

$$Mc = \frac{dc}{dx} \text{ ------------- (5)}$$

Since, we know that integration is reverse process of differentiation that is if $\frac{d}{dx} f(x) = F(x)$ then

$\int F(x)dx=f(x)+c,$ where C is an arbitrary constant.

Therefore, if we integrate equation (5) on both sides we got

$$C(x)= \int (MC)\, dx+c \text{ -------------- (6)}$$

We know that

$$R(x) = P \times x \text{ ---------------- (7)}$$

If differentiate equation (7), we get the rate of change in revenue per unit in out, which is called as marginal revenue (MR) and gives as –

$$MR = \frac{dR}{dx} \text{ ---------------- (8)}$$

Taking integration of equation (8) on both sides with respect to x, $R(x) = f(MR)dx+c,$ where c is arbitrary constant ………………. (9)

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$$\pi(x) = R(x) – C(x) \text{ ------------- } (10)$$

If the equation (10) is differentiated with respect to x, we get marginal profit

$$(MP) = \frac{dp}{dx} \text{ -------------- } (11)$$

We know that □ = $-\frac{p}{x}\frac{dx}{dp}$ Where e_d = Elasticity of demand

The value of ed is described as below,

If demand is elastic if ed>1

If demand is inelastic if ed<1

If demand is unitary if ed=1

Applications

Following are the applications of calculus in economicsas shown in below.

Application 1 (Marginal cost Analysis): -The total cost in thousands of rupees for a daily production of an item is-

$c(x) = 50 + 20x – x2$ then

$MC = \frac{d}{dx} 50 + 20x – x^2 = 20\text{-}2x$

When 4 units are produced is given by

$$MC = 20–2(4) = 20–8 = 12 \text{ thousand}$$

Application 2 (Total Revenue):- Marginal revenue of a function is

$MR = 7 – 2x – 3x^2$ then

$$TR = \int (7 – 2x – 3x^2)\, dx = 7x – \frac{2x^2}{2} – \frac{3x^3}{3}$$

$$= 7x – x^2 – x^3$$

Application 3 (Tangent line to cost function curve):- Suppose that, the cost to produce x units of a particular item is

$$C(x) = 1.25x + 0.01x^2 + 50$$

$$MC = 1.25 + 0.02x$$

If, $P = 3$ then, $R(x) = P.x = 3x$

We know that, $\pi = R(x) – C(x)$ or $3x – 1.25x – 0.01x^2 – 50$

$$\text{or, } 1.75x – 0.01x^2 – 50$$

$$\pi'(x) = 1.75 – 0.02x$$

50 is a fixed cost that must be paid to produce any items.

Application 4 (Economic scale):- The Average cost per unit is defined as-

$$AC = \frac{c}{x}$$

If, the cost function for any is $c = 500 + 35x$ then $AC = \frac{500}{x} + 35$

and $\frac{d}{dx}(AC) = \frac{(-500)}{x^2}$, Since the derivative of the average cost is negative for all, we see that the average cost is decreasing for all x.

Application 5 (Elasticity of demand):- Elasticity of demand is determined by-

$$e_d = \frac{-p}{x} \cdot \frac{dx}{dp}$$

If demand function x = 76 – 3p, then, ed= ?

We know that, $e_d = \frac{-p}{x} \cdot \frac{dx}{dp}$

x = 76 – 3p

dx / dp = - 3

ed = [- p/ (76-3p)] . (-3)

= 3p / (76-3p)

If, p = 20 then ed = 3 X 20 / (76 – 3 X 20) = 60/16 = 15/4

Application 6 : A company sells shoes to dealers at $20 per pair if fewer than 50 pairs are ordered. If 50 or more pairs are offered (up to 600), the price per pair is reduced 2 percent times the number ordered. What size orders produce maximum revenue for the company?

$$R = p.x = (20x) \text{ } if \text{ } x \in (0,50)$$

Taking the quantity discount into consideration, we get

$$R(x) = (20 - 0.02x)\, x \text{ } if \text{ } x \in (51,60)$$

When $R(x) = 20x$ and it is obvious that revenue is max. when $x = 50$

$$\therefore R = 20.50 = 1000\$$$

$$R(x) = 20x - 0.02x^2$$

$$R'(x) = 20 - 0.04x$$

R″ (x) = – 0.04

The critical number for revenue is $\frac{20}{0.04}$ = 500 and since R″ (x) < 0 for all x, we know that R(x) has a max. at $x = 500$

$$\therefore R(x) = 20 \times 500 - 0.02\,(500)^2$$

$$= 10000 - 0.02 \times 250000$$

$$= 5000\$$$

$$\therefore R(500) > R(50)$$

So, we know that the order size producing the most income for the company is a 500-pair order Now, we see the application of integration in economics. The supply function gives the quantity of an item that procures and will supply at any given price. The demand function gives the quantity that consumers will demand at any given price. Price per unit represents by p and the quantity supplied or demanded at price represent by q. The supply and demand curve is demoted by the formula.

$$P = S(q) \text{ -------------- } (13)$$

$$P = D(q) \text{ -------------- } (14)$$

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As you might expect, the supply function S is increasing the higher the price, the more the producers will supply. The demand function D is decreasing- the higher the price, The less the consumers will buy, The point of intersection (*qc, pc*) of the supply and demand curve is called the market equilibrium point.

Total amount spent at equilibrium price $P = qc\text{X}pc$ ------------ (15) Continuing this process of price discrimination, the total amount of money paid by consumers willing to pay at least p_c is approximately equal to –

$D(x_1)\,\Delta x + D(x_2)\,\Delta x + D(x_3)\,\Delta\text{x} + \ldots \ldots\ldots\ldots\; D(x_n)\,\Delta k$ ------------- (16)

The total amount of the maximum price is given by $\int_0^{qc} D(q)dq$ ----------- (17)

From the equation, (15) and (17), we have two areas (17)-(15) represents the total consumers save by buying at an equilibrium price. This is called the consumer surplus for this product and is given by-

$$C.S = \int_0^{qc} D(q)dq - p_c\, q_c \text{ ----------- (18)}$$

qc

Similarly, the producer surplus is given by-

$$P.S. = p_c\, q_c - \int_0^{qc} \Box\, S(q)dq \text{ ------------ (19)}$$

Application 7(Maximizing Revenue):- The demand equation for a certain product is $P = 6 - \frac{1}{2}q$

$$R = p.q = (6 - \tfrac{1}{2}q).\ q = 6q - \tfrac{1}{2}q^2$$

$R'(q) = MR = 6 - \frac{1}{2}.2q = 6 - q \quad \therefore\ q = 6$ which is the criticl point.

$R'(q) = -1 < 0$, hence we get max. value at $q = 6$ and $R = 6.6 - \frac{1}{2}b2 = 18$

Application 8(Use of Integral in Economics): -

Calculus is used to determine consumer surplus and producer surplus.

Consumer surplus:

If the demand law is p = 85 – 4x – x2, what will be the consumer surplus? If, p0 = 64

p = 85 – 4x – x2

64 = 85 – 4x – x2

or, x2 + 4x – 85 + 64 = 0

or, x2 + 4x – 21 = 0

or, x2 + 7x – 3x – 21 = 0

or, x (x+7) – 3 (x+7) = 0

or, (x-3) (x+7) = 0

If, x + 7 = 0, therefore, x = -7 is not acceptable

If, x -3 = 0, therefore, x = 3 is acceptable

Therefore, x0 = 3

R = P.x = 64 x 3 = 192

C.S. = $\int_0^3 P\,dx - R$

= $\int_0^3 (85 - 4x - x^2)\,dx - R$

= $85 \int_0^3 dx - 4 \int_0^3 x dx - \int_0^3 x^2 dx - 192$

= 85 x 3 – 4 x(9/2) – 27 / 3 – 192

= 255 – 18 – 9 – 192

= 255 – 219 = 36

Producer surplus:

If pd = 3x2 – 20 x + 5 and ps = 15 + 9x, then P.S. will be-

In equilibrium,

Pd = ps

3x2 – 20 x + 5 = 15 + 9x

3x2 – 20 x – 9x + 5 – 15 = 0

3x2 – 29 x– 10 = 0

3x2 – 30x + x – 10 = 0

3x (x-10) + 1 (x – 10) = 0

(3x+1) (x-10) = 0

If, 3x + 1 = 0, therefore, x = –1/3 is not acceptable

If, x – 10 = 0, therefore, x = 10 is acceptable

Therefore, x0 = 10

Ps = 15 + 9x

= 15 + (9 X 10)

= 105

R = P.x = 105 X 10 = 1050

P.S. = R – $\int_0^{10} P\,dx$

= 1050 – $\int_0^{10} dx + 9 \int_0^{10} x dx$

= 1050 – [15 X 10 + (9 X 100/2)]

= 1050 – [150+450]

= 1050 – 600 = 450

Conclusion

With Integration and differentiation, as mathematical techniques, I have been able to reach the solution of the different methods used. Each of the methods brings a simple and clear way and thus can be used for practical data. Furthermore, with figures, I have been able and compare the actual data with the predicted data.

By determining marginal revenue and cost can help business managers maximize their profits and measure the rate of increase in profit that results from each increase in production. As long as marginal revenue exceeds marginal cost, the farm increases its profit.

The use of calculus in economics plays a vital role in demand and supply theory. The prices of an item determine the supply and demand. As the price increases, demand for the item usually falls. But as the price increases, the producers will increase the supply. In nutshell, we say that, the application problems, marginal analysis and cost analysis can be computed more easily.

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